# High-Field Effects in Degenerate Nonparabolic Semiconductors: Collision Integral and Isotropic Distribution Function

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Collision integrals for various modes of lattice scattering for nonparabolic semiconductors with an arbitrary degree of degeneracy of charge carriers have been derived, taking Pauli's exclusion principle into account. These expressions are used to obtain the isotropic part of the momentum distribution function of transport carriers subjected to a dc field for different cases of interest. This formulation avoids the use of the concept of effective carrier temperature in the calculation of transport and other properties of arbitrarily degenerate semiconductors. It is pointed out that the assumed form of the distribution function with effective carrier temperature and drift velocity in the case of parabolic piezoelectric semiconductors, which is used to investigate acoustic-wave amplification in the presence of high dc fields, is not justified for a wide upper range of applied dc fields. This is so because above a certain critical value of the dc field the distribution function obtained by solving the Boltzmann transfer equation is not normalizable. It is shown that the distribution function corresponding to the piezoelectric mode of lattice scattering can be normalized for any finite value of the applied dc field if the nonparabolicity of energy bands is taken into account (apart from the influence of nonequilibrium phonons on the normalization of the carrier distribution function).

## I. INTRODUCTION

In an earlier communication[1] (which will be referred to as I) it is emphasized that the knowledge of the collision term in the Boltzmann transfer equation is essential for an adequate analytical investigation of transport phenomena and other properties of semiconductors, particularly in the presence of high dc fields. In the absence of the knowledge of this collision term, the hot-carrier phenomena in degenerate semiconductors have been analyzed by using the concept of effective carrier temperature.[2–5] This approach has led to many interesting results, but suffers from the weakness of the basic assumption that the form of the isotropic part of the distribution function remains unchanged on the application of an electric field. In the case of nondegenerate simple-model semiconductors, the results obtained on the assumption of effective carrier temperature are at considerable variance with those obtained by solving the Boltzmann transfer equation with the corresponding collision term.[6] In I the authors have presented a derivation of collision integrals for different modes of lattice scattering for semiconductors with parabolic energy bands, taking into account Pauli's exclusion principle and hence, degeneracy.

It is well known that the conduction bands of many semiconductors of immense practical utility (e.g., III-V[7,8] and II-VI[9] compounds) are nonparabolic. Nonparabolic energy-momentum relationships are shown to have a very significant influence on the magnitude,[8,10,11] and in some cases, even on the sign of the transport coefficients.[12] Hence, in this paper the authors have derived the collision operators and isotropic parts of the carrier energy distribution function corresponding to various modes of lattice scattering, taking into account Pauli's exclusion principle (i.e., degeneracy) and the realistic nonparabolic energy-band structure.

The Maxwell-Boltzmann (MB) distribution function is obtained from the Fermi-Dirac (FD) distribution function by making use of the fact that the Fermi level lies much below ($>4K_0T$) the bottom of the conduction band in nondegenerate semiconductors, and hence many of the properties of nondegenerate semiconductors are independent of carrier concentration. In fact, the effectiveness of the approximation for applicability of MB distribution to fermions (charge carriers) is also determined by the energy dependence of the quantity for which we need an average over the distribution function. The considerations of nonparabolicity introduce additional energy dependances. It is therefore desirable that Pauli's exclusion principle be included in the derivation of distribution function in the case of nonparabolic semiconductors, even for the cases where the Fermi level lies $4K_0T$ or more below the bottom of the conduction band.

Laikhtman[13] has investigated the distribution function of carrier energy in the nondegenerate simple-model semiconductors. He has concluded that the distribution function can not be normalized for dc fields higher than some critical value determined by the parameters of the medium which is about 1 V/cm. at 4 °K and about 50 V/cm at room temperatures for a typical piezoelectric semiconductor. It is experimentally well established that the carriers do not escape out of the sample when dc fields exceeding the above mentioned fields are applied to piezoelectric semiconductors to impart supersonic velocities to the carriers for obtaining amplification of the acoustic waves.[14] The theoretical analyses of acoustic wave amplification in piezoelectric semiconductors due to carrier drift are based on either the elementary approach[15,16] or on an assumed form of the isotropic part of distribution function.[17–20] The isotropic part of the carrier distribution function has not been obtained by solving equations for electrons and phonons for the hot-electron case. The assumed form of the distribution function is not at all justified in the range of fields where the distribution function can not be normalized even if we allow for the use of the concept of effective carrier temperature and drift velocity. It is shown in the present paper that the carrier momentum distribution function in a piezoelectric semiconductor can be normalized for all finite values of the externally applied dc field if the nonparabolicity of energy bands[8,10,21] is taken into account, irrespective of the extent of degeneracy of the charge carriers. Similar results for nondegenerate semiconductors have been obtained for the polar optical mode of the lattice scattering by Matz[22] and Dykmann and Tomchuk.[23]

We present in Sec. II the general derivation of the collision operator taking into account Pauli's exclusion principle and nonparabolicity of the energy bands. The inelasticity of the carrier lattice interaction is explicitly taken into account. To facilitiate the derivation of the collision operator, the high-temperature approximation has been used. In Sec. III we obtain the collision integrals and the distribution function (in the presence of a dc field) corresponding to various modes of the lattice scattering. In Sec. IV we briefly summarize the conclusions drawn in this paper.

## II. COLLISION OPERATOR

The general expression for the rate of change of the distribution function $f_{\vec{k}}$ due to the interaction between carriers and the lattice given by Conwell[24] may be modified so as to include Pauli's exclusion principles; thus following I one obtains

$$\begin{aligned}\frac{\partial f(\vec{k})}{\partial t} = \frac{2\pi}{\hbar}\sum_{\vec{q}}\Big[&|(\vec{k},N_{\vec{q}}+1|H'|\vec{k}+\vec{q},N_{\vec{q}})|^2\delta(E_{\vec{k}}-E_{\vec{k}+\vec{q}}+\hbar\omega_{\vec{q}})f(\vec{k}+\vec{q})]\cdot[1-f(\vec{k})]\\ &+|(\vec{k},N_{\vec{q}}-1|H'|\vec{k}-\vec{q},N_{\vec{q}})|^2\delta(E_{\vec{k}}-E_{\vec{k}-\vec{q}}-\hbar\omega_{\vec{q}})f(\vec{k}-\vec{q})\cdot[1-f(\vec{k})]\\ &-|(\vec{k}-\vec{q},N_{\vec{q}}+1|H'|\vec{k},N_{\vec{q}})|^2\delta(E_{\vec{k}-\vec{q}}-E_{\vec{k}}+\hbar\omega_{\vec{q}})\cdot f(\vec{k})\cdot[1-f(\vec{k}-\vec{q})]\\ &-|(\vec{k}+\vec{q},N_{\vec{q}}-1|H'|\vec{k},N_{\vec{q}})|^2\delta(E_{\vec{k}+\vec{q}}-E_{\vec{k}}-\hbar\omega_{\vec{q}})f(\vec{k})\circ[1-f(\vec{k}+\vec{q})]\ ,\end{aligned} \tag{1}$$

where $\vec{k}$ and $\vec{q}$ are the electron and phonon wave vectors, respectively, and the quantity $(2\pi/\hbar)|(\vec{k}'|H'|\vec{k})|^2 \times\delta(E_1-E_2)$ gives the probability per unit time that a carrier is scattered in the state $\vec{k}'$ from its initial state $\vec{k}$. Thus the four terms on the right-hand side of Eq. (1) correspond, respectively, to the following: (i) The carrier may be scattered into the state $\vec{k}$ from a state $\vec{k}+\vec{q}$ by the emission of a phonon. (ii) The carrier may be scattered into the state $\vec{k}$ from a state $\vec{k}-\vec{q}$ by absorption of a phonon. (iii) The carrier may be scattered out of the state $\vec{k}$ into a state $\vec{k}-\vec{q}$ by emission of a phonon. (iv) The carrier may be scattered out of the state $\vec{k}$ into a state $\vec{k}+\vec{q}$ by absorption of a phonon. $N_{\vec{q}}$ is the equilibrium phonon distribution. We convert the summation over $\vec{q}$ to an integration over $\vec{q}, \theta$, and $\phi$ in Eq. (1) and obtain

$$\begin{aligned}\left(\frac{\partial f}{\partial t}\right)_c = \frac{2\pi}{\hbar}\frac{2V}{(2\pi)^3}\Bigg\{&\int_0^{2\pi}d\phi\int_0^{\pi}\sin\theta\, d\theta\int_{q'_{\min}}^{q'_{\max}}C(\vec{q})N_{\vec{q}}[f(\vec{k}+\vec{q})\cdot(1-f(\vec{k}))\exp\chi_{\vec{q}}-f(\vec{k})(1-f(\vec{k}+\vec{q}))]\delta(\chi_{\vec{k}+\vec{q}})q^2dq\\ &+\int_0^{2\pi}d\phi\int_0^{\pi}\sin\theta\, d\theta\int_{q''_{\min}}^{q''_{\max}}C(\vec{q})N_{\vec{q}}[f(\vec{k}-\vec{q})\cdot(1-f(\vec{k}))-f(\vec{k})(1-f(\vec{k}-\vec{q}))\exp\chi_{\vec{q}}]-\delta(\chi_{\vec{k}-\vec{q}})\,q^2dq\Bigg\}\ ,\end{aligned} \tag{2}$$

where $C_{\vec{q}}$ describes the transition probability due to the electron-phonon interaction and depends upon the lattice mode, $\chi_{\vec{q}}=\hbar\omega_{\vec{q}}/k_0T$, $\chi_{\vec{k}\pm\vec{q}}=E_{\vec{k}}-E_{\vec{k}\pm\vec{q}}\mp\hbar\omega_{\vec{q}}$, and $V$ is the crystal volume. The limits of integration $q'^{,\prime\prime}_{\max,\min}$ over $q$ are obtained by considering the fact that the argument of the $\delta$ function should vanish in the entire range of integration. The integration over $\theta$ in Eq. (2) is replaced by an integration over the energy. For further analysis, we consider the spherical energy surfaces and the following relationship between wave vector and energy of the carrier due to

Kane[7]:

$$\gamma_{\vec{k}} = \hbar k^2/2m = E_k(1 + E_k/E_g) \quad , \tag{3}$$

where $m$ is the band-edge mass, and $E_g$ is the energy gap. For parabolic energy bands $\gamma_{\vec{k}} = E_{\vec{k}}$ (obtained by letting $E_g \to \infty$). According to Eq. (3) we have

$$\gamma_{\vec{k}\pm\vec{q}} = \gamma_{\vec{k}} + \gamma_{\vec{q}} \pm (\hbar^2/m)\, kq \cos\theta \quad , \tag{4}$$

and on differentiating Eq. (4) we obtain

$$\sin\theta\, d\theta = \mp \left(\frac{m}{\hbar^2 kq}\right)\left(\frac{d\gamma_{\vec{k}\pm\vec{q}}}{dE_{\vec{k}\pm\vec{q}}}\right) dE_{\vec{k}\pm\vec{q}} \quad . \tag{5}$$

As discussed by Kolodziejaczak,[25] the integration over $\theta$ in Eq. (2) can be replaced by an integration over $E_{\vec{k}\pm\vec{q}}$ with the new limits of integration $\pm L$ where $L = \hbar^2 kq/m$. Because of the properties of the δ function these limits can be replaced by $\pm\infty$.

We consider the case of externally applied dc fields for which the drift velocity of carriers is much smaller than their thermal velocity, and hence we make the following expansion of the distribution function:

$$f(\vec{k}) = f_0(E_{\vec{k}}) + k_F\, g(E_{\vec{k}}) \quad , \tag{6}$$

where $k_F$ is the carrier wave vector in the direction of the dc field and $f_0$ and $k_F g$ are the isotropic and anisotropic parts of the distribution function, respectively. We substitue Eqs. (5) and (6) into Eq. (2) and make use of the high-temperature approximation, i.e., $\hbar\omega_{\vec{q}} \ll k_0 T$ and also $\hbar\omega_{\vec{q}} \ll E_{\vec{k}}$, since at high temperatures very slow electrons for which $E_k \leq \hbar\omega_{\vec{q}}$ make a negligible contribution to the transport processes; thus we may expand $f_0(\vec{k}\pm\vec{q})$, $g(\vec{k}\pm\vec{q})$, and $\exp\chi_{\vec{q}}$ in the power series of $\hbar\omega_{\vec{q}}$. We finally obtain the following collision term:

$$\begin{aligned}\left(\frac{\partial f_0}{\partial t}\right)_c = \frac{mk_0TV}{2\pi\hbar^3 k}\Bigg[&\left(f_0' + \frac{f_0(1-f_0)}{k_0T}\right)\left(\int_{q'_{\min}}^{q'_{\max}} C(q)\,dq - \int_{q''_{\min}}^{q''_{\max}} C(q)\,dq\right)\\ &+\left(\frac{d\gamma}{dE}\right)^{-1}\frac{d^2\gamma}{dE^2}\left(f_0' + \frac{1}{k_0T} f_0(1-f_0)\right)\left(\int_{q'_{\min}}^{q'_{\max}} qC(q)\,dq + \int_{q''_{\min}}^{q''_{\max}} qC(q)\,dq\right)\\ &+\frac{1}{2}\left(f_0'' + \frac{2}{k_0T} f_0'(1-f_0)\right)\int_{q'_{\min}}^{q'_{\max}} q\hbar\omega_{\vec{q}}\, C(q)\,dq\\ &+\frac{1}{2}\left(f_0'' - \frac{2}{k_0T} f_0' f_0\right)\int_{q''_{\min}}^{q''_{\max}} q\hbar\omega_{\vec{q}}\, C(q)\,dq\Bigg]\end{aligned} \tag{7a}$$

and

$$\frac{\partial}{\partial t}(k_F g) = -k_F g\, \frac{mk_0TV}{4\pi\hbar^4 k^3}\,\frac{d\gamma}{dE}\left(\int_{q'_{\min}}^{q'_{\max}} \frac{q^3}{\omega_{\vec{q}}}\, C(q)\,dq + \int_{q''_{\min}}^{q''_{\max}} \frac{q^3}{\omega_{\vec{q}}}\, C(q)\,dq\right) , \tag{7b}$$

where the primes denote differentiation with respect to energy $E$.

## III. DISTRIBUTION FUNCTION

The Boltzmann transfer equation for the carrier momentum distribution function is

$$\left(\frac{\partial f}{\partial t}\right)_{\text{field}} + \left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = 0 \quad , \tag{8}$$

where $(\partial f/\partial t)_{\text{field}}$ is written as usual[22] as

$$\left(\frac{\partial f}{\partial t}\right)_{\text{field}} = \frac{eF}{\hbar y'}\left(\frac{\hbar^2 k_F}{mk_0T} f_0' + \tfrac{2}{3} y^{-1/2}\frac{d}{dx}(gy^{3/2})\right) , \tag{9}$$

where $-e$ is the charge of electron, $F$ is the externally applied dc field, $x = E/k_0T$, $y = \gamma/k_0T$, and the primes denote differentiation with respect to the dimensionless energy $x$.

### A. Acoustic-Mode Scattering

In the case of acoustic modes we have[26]

$$\omega_{\vec{q}} = u_l q \quad \text{and} \quad C(q) = E_1^2\, \hbar\omega_{\vec{q}}/2\rho V u_l^2 \quad , \tag{10a}$$

where $u_l$ is the velocity of sound, $E_1$ is the band-edge shift per unit dilation, and $\rho$ is the density of the crystal.

The limits of integration in the Eqs. (7a) and (7b) are obtained in Ref. 25 as follows:

$$q'_{\min} = q''_{\min} = 0 \quad ,$$

$$q'_{\max} = 2k + \frac{2mu_l}{\hbar}\, y' \quad \text{and} \quad q''_{\max} = 2k - \frac{2mu_l y'}{\hbar} \quad . \tag{10b}$$

Substituting Eqs. (10a) and (10b) into Eqs. (7a) and (7b), we obtain the following collision operator:

$$\left(\frac{\partial f}{\partial t}\right)_c = C_a y^{-1/2} y'^{-1} \frac{d}{dx}\{y^2 y'^2[f_0' + f_0(1-f_0)]\} - \nu_{0a} y^{1/2} y' k_F g \, , \tag{11}$$

where

$$C_a = 2\sqrt{2}\, E_1^2 m^{5/2}(k_0T)^{1/2}/\pi\rho\hbar^4$$

and

$$\nu_{0a} = \sqrt{2}\, E_1^2 m^{3/2}(k_0T)^{3/2}/\pi\rho\hbar^4 u_l^2 \, .$$

It is seen from Eq. (11) that the collision operation corresponding to the isotropic part of the distribution function vanishes if we substitute the equilibrium distribution function $f_0 = 1/(1+e^{x-\eta})$ (as expected), where $\eta$ is the dimensionless Fermi energy $E_F/k_0T$.

We substitute Eqs. (11) and (9) into Eq. (8) and obtain the following two equations after separating isotropic and anisotropic parts:

$$C_a\{y^2 y'^2[f_0' + f_0(1-f_0)]\} + \tfrac{2}{3}(eF/\hbar) y^{3/2} g = 0 \tag{12a}$$

and

$$\nu_{0a} y^{1/2} y' g = eF\hbar/mk_0T\, y'^{-1} f_0' \, . \tag{12b}$$

After eliminating $g$ between Eqs. (12a) and (12b) we obtain

$$yy'^4[f_0' + f_0(1-f_0)] + p_a f_0' = 0 \, , \tag{13}$$

with the solution

$$f_0 = (1 + e^{x-\eta-I_1})^{-1} \, ,$$

where

$$I_1 = p_a \int dx/(1+e^{x-\eta-I_1}) \, , \qquad p_a = \tfrac{2}{3} e^2F^2/\nu_{0a} C_a m k_0 T \, , \tag{14}$$

and $\eta$ is the dimensionless Fermi energy when $p_a = 0$. For the nondegenerate case we may let $\eta \to -\infty$. In addition, for the parabolic nondegenerate case we may put $y = x$. It is then seen that for a simple-model nondegenerate semiconductor the expression for $f_0$, given by Eq. (14), reduces to $f_0 = (x+p_a)^{p_a} e^{-x}$, given by Yamashita and Watanabe[27] and with the same substitution, Eq. (11) reduces to the corresponding equation given by those authors.[27]

### B. Piezoelectric-Mode Scattering

In the case of the piezoelectric mode of lattice scattering we have[28]

$$\omega_{\vec{q}} = u_l q \quad \text{and} \quad C(q) = 8\pi^2\hbar e^2\beta^2 u_l^2/\rho V \epsilon_s^2 \omega_q \, , \tag{15}$$

where $\epsilon_s$ is the dielectric constant and $\beta$ is the piezoelectric constant assumed to be isotropic for simplicity. Because of the $q$ dependence of $\omega_{\vec{q}}$ as in the case of an acoustic mode, the limits of integration in Eqs. (7a) and (7b) are given by Eq. (10b). Substituting Eqs. (15) and (10b) into Eqs. (7a) and (7b), we obtain the following collision operator:

$$\left(\frac{\partial f}{\partial t}\right)_c = C_{pe} y^{-1/2} y'^{-1} \frac{d}{dx}\{yy'^2[f_0' + f_0(1-f_0)]\} - \nu_{0pe} y^{-1/2} y' k_F g \, , \tag{16}$$

where

$$C_{pe} = 16\sqrt{2}\, \pi m^{3/2} u_l^2 e^2\beta^2/\rho\epsilon_s^2 \hbar^2 (k_0T)^{1/2}$$

and

$$\nu_{0pe} = 8\sqrt{2}\, \pi m^{1/2} e^2\beta^2 (k_0T)^{1/2}/\rho\epsilon_s^2 \hbar^2 \, .$$

We substitute Eqs. (16) and (9) into Eq. (8) and obtain two equations after separating isotropic and anisotropic parts of the resulting equation. After eliminating $g$ between these two equations we obtain

$$y'^4[f_0' + f_0(1-f_0)] + p_e y f_0' = 0 \, , \tag{17}$$

with the solution

$$f_0 = (1 + e^{I_2 - \eta})^{-1} \, , \tag{18}$$

where

$$I_2 = \int [\, y'^4/(y'^4 + p_e y)]\, dx$$

and

$$p_e = \tfrac{2}{3} e^2F^2/\nu_{0pe} C_{pe} m (k_0T) \, .$$

For the case of nondegenerate parabolic semiconductors, $f_0$ given by Eq. (18) reduces to that given by Laikhtman[13] and is as follows:

$$f_0 = (x + 1/p_e)^{-1/p_e} . \tag{19}$$

As discussed by Laikhtman,[13] $f_0$ given by Eq. (19) cannot be normalized for fields given by

$$p_e > 1/(960\pi B) \, , \tag{20}$$

where $B \sim 1$. For a typical piezoelectric semiconductor the critical value of field strength is about 1 V/cm at 4 °K and 50 V/cm at 300 °K.

From Eqs. (18) and (18a) it is seen that $\int_0^\infty x^{1/2} f_0 \times dx$ is finite for any value of $p_e$ and $\eta$. Thus, for the case of nonparabolic energy bands expressed by Eq. (3) the carrier momentum distribution function for piezoelectric semiconductors can be normalized for any finite value of the externally applied dc fields and for arbitrary degeneracy of charge carriers. The effectiveness of the piezoelectric mode of lattice scattering is determined by the average energy of charge carriers through energy dependence of the corresponding collision frequency at any value of the externally applied dc field.

The normalizability of the distribution function because of the band nonparabolicity may be interpreted as follows. The effective mass of the carrier increases with its energy; effective mass becomes infinite for infinite energy of the carrier. An increase in the effective mass implies corresponding decrease in the effective velocity[29] of the carrier with a particular energy. We expect the distribution function to have maxima at some particular effective velocity of carriers and then it goes on decreasing with further increase in the effective velocity. By normalizability of the distribution function we mean that there are no carriers with infinite effective velocities. The influence of the band nonparabolicity is to enforce an effective velocity distribution which is normalizable – the area under the curve of the distribution function versus the effective carrier velocity is finite.

In the present investigation the phonon distribution function is assumed to be in thermal equilibrium. Under the conditions of acoustic-wave amplification, an appreciable phonon flux is generated. The considerations of phonon flux generation may also lead to normalization of the carrier velocity distribution function apart from the consideration of band nonparabolicity undertaken here.

### C. Polar Optical-Mode Scattering

In the case of the polar mode of lattice scattering, we have[26]

$$\omega_{\vec{q}} = \omega_l \quad \text{and} \quad C(q) = 2\pi\hbar^2 eE_0/Vmq^2 , \tag{21a}$$

where

$$eE_0 = (me^2\hbar\omega_l/\hbar^2)[(1/\epsilon_\infty) - (1/\epsilon_0)] ;$$

$\omega_l$ is the longitudinal optical-mode frequency, and $\epsilon_\infty$ and $\epsilon_0$ are high-frequency and static dielectric constants of the medium. The limits of integration in the Eqs. (7a) and (7b) are obtained in Ref. 25 as follows:

$$q'_{\min} = q''_{\min} = (y'/2)(\hbar\omega_l k/\gamma)$$

and

$$q'_{\max} = 2k + q'_{\min} , \quad q''_{\max} = 2k - q''_{\min} . \tag{21b}$$

Substituting Eqs. (21a) and (21b) into Eqs. (7a) and (7b), we obtain the following collision operator:

$$\left(\frac{\partial f}{\partial t}\right)_c = C_{p0} y^{-1/2} y'^{-1} \frac{d}{dx}\{y'^2[f_0' + f_0(1-f_0)]\ln(4\gamma_0)\}$$
$$- \nu_{0p0}\, y^{-1/2} y' k_F g , \tag{22}$$

provided $x_0 \ll 1$ (high-temperature approximation), and where

$$C_{p0} = eE_0 x_0^2 (2mk_0T)^{-1/2}(e^{\Theta/T} - 1)^{-1} ,$$

$$\nu_{0p0} = \sqrt{2} eE_0 (mk_0T)^{-1/2}(e^{\Theta/T} - 1)^{-1} ,$$

$$Y_0 = y x_0^{-1} y'^{-1} , \qquad x_0 = \hbar\omega_l/k_0T ,$$

and $\Theta$ is the Debye temperature.

From the Eqs. (22), (9), and (8) we obtain the following equation:

$$y'^4 \ln(4Y_0)[f_0' + f_0(1-f_0)] + p_{p0} y^2 f_0' = 0 , \tag{23}$$

with the solution

$$f_0 = (1 + e^{I_3 - \eta})^{-1} , \tag{24}$$

where

$$I_3 = \int \frac{dx}{1 + \frac{2}{3}(F/E_0)^2 y^2/[y'^4 \ln(4Y_0)]}$$

and

$$p_{p0} = \tfrac{2}{3}(F/E_0)^2 .$$

For the nondegenerate nonparabolic semiconductors, $f_0$ given by Eq. (24) reduces to that given by Matz[22] and Dykman and Tomchuk[23]

### D. Nonpolar Optical-Mode Scattering

In the case of nonpolar optical mode of lattice scattering we have[26]

$$\omega_{\vec{q}} = \omega_0 \quad \text{and} \quad C(q) = E_{10p}^2 \hbar\omega_0/2V\rho u_l^2 , \tag{25}$$

where $\omega_0$ is the frequency of the nonpolar optical mode of lattice vibration. The limits of integration in the Eqs. (7a) and (7b) for this case are those given by Eq. (21b) if $\omega_l$ is replaced by $\omega_0$. Substituting these limits and Eq. (25) in the Eqs. (7a) and (7b) we obtain the following collision operator:

$$\left(\frac{\partial f}{\partial t}\right)_{0p} = C_{0p}' y^{-1/2} y'^{-1} \frac{d}{dx}\{y y'^2[f_0' + f_0(1-f_0)]\}$$
$$- \nu_{0p}' y^{1/2} y' k_F g ,$$

provide $x_0 \ll 1$ and where

$$C_{0p}' = m^{3/2} E_{10p}^2 \omega_0^2 / 2\sqrt{2}\pi\hbar^2 \rho u_l^2 (k_0T)^{1/2} ,$$

$$\nu_{0p}' = m^{3/2}(k_0T)^{3/2} E_{10p}^2/\sqrt{2}\pi\hbar^4 \rho u_l^2 ,$$

$$E_{10p}^2 = D_t^2 K^2 u_l^2/\omega_0^2 , \qquad x_0 = \hbar\omega_0/k_0T ,$$

$K$ is the first reciprocal vector of lattice, and $D_t$ is the coupling constant between electrons and nonpolar optical mode of lattice vibration. From the Eqs. (26), (9), and (8) we obtain the following equation:

$$y'^4[f_0'+f_0(1-f_0)]+p_{0p}f_0'=0\ ,$$

with solution

$$f_0=(1+e^{I_4-\eta})^{-1}\ ,$$

where

$$I_4=\int dx/(y'^4+p_{0p})$$

and

$$p_{0p}=2e^2F^2/3\nu_{0p}'\,C_{0p}'\,m_0T$$

The distribution function corresponding to any case of mixed scattering can be obtained by using the above derived collision integrals and following the standard techniques.[1] A derivation of Brooks-Herring formula[30] for ionized impurity scattering including explicitly the nonparabolicity of energy bands will be presented separately.

## IV. CONCLUSIONS

We have derived the expressions for collision integrals corresponding to various modes of the lattice scattering taking into account Pauli's exclusion principle and the realistic nonparabolic energy-band structure particularly appropriate to III – V semiconductors. The energy surfaces are assumed to be spherical. These expressions are used to obtain the carrier momentum distribution function in the presence of a dc field for the case of arbitrary degeneracy of charge carriers. This formulation avoids the use of the concept of effective carrier temperature in the calculation of transport properties of semiconductors with arbitrary degeneracy of the charge carriers and nonparabolic energy bands.

It may be pointed out that the usually assumed form of the electron velocity distribution function (i.e., a Fermi-Dirac distribution with an effective carrier temperature), which is employed in many investigations, is not justified. It is because of this fact that the distribution function of carrier velocities in simple-model piezoelectric semiconductors obtained by solving the Boltzmann transfer equation in the presence of dc fields with appropriate collision term can not be normalized for fields greater than some critical value. It is shown that for the above case the distribution function can be normalized for any finite value of applied dc fields if the nonparabolicity of energy bands is taken into account apart from the influence of nonequilibrium phonons.

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[1]M. S. Sodha and B. M. Gupta, Phys. Rev. (to be published).

[2]R. F. Green, J. Electron. Control 3, 387 (1957).

[3]A. Zylbersztejn, *Proceedings of the Seventh International Conference on the Physics of Semiconductors*, (Academic, New York, 1965), p. 505.

[4]L. E. Gurevich and I. Ya. Korenblit, Zh. Eksperim. i Teor. Fiz. 44, 2150 (1963) [Soviet Phys. JETP 17, 1444 (1963)].

[5]J. Kamal, Phys. Status Solidi, 35, 801 (1969).

[6]E. M. Conwell, *High Field Transport in Semiconductors*, (Academic, New York, 1967), p. 11-4.

[7]O. E. Kane, J. Phys. Chem. Solids 1, 249 (1957).

[8]L. Sosnowski, *Proceedings of the International Conference on the Physics of Semiconductors, Paris*, 1964 (Dunod, Paris, 1964), p. 341.

[9]J. O. Dimmock, *Proceedings of the International Conference on* II–VI *Semiconducting Compounds, Providence*, 1967, edited by D. G. Thomas (Benjamin, New York, 1967), p. 277.

[10]N. N. Grigorev, I. M. Dykman, and P. M. Tomchuk, Soviet Phys. Semicond. 1, 101 (1967).

[11]T. S. Statvitskaya, L. V. Prokofe'va, Yu. I. Ravich, and B. A. Efimova, Soviet Phys. Semicond. 1, 952 (1967).

[12]J. Kolodziejczak and L. Sosnowski, Acta Phys. Polon. 21, 399 (1962).

[13]B. D. Laikhtman, Fiz. Tverd. Tela 6, 3217 (1964) [Soviet Phys. Solid State 6, 2573 (1965)].

[14]A.-R. Hutson, J. H. McFee, and D. L. White, Phys. Rev. Letters 7, 237 (1961).

[15]P. K. Tien, Phys. Rev. 171, 970 (1968).

[16]D. L. White, J. Appl. Phys. 33, 2547 (1962).

[17]H. Spector, Phys. Rev. 127, 1084 (1962).

[18]J. Yamashita and K. Nakamura, J. Phys. Soc. Japan Suppl. 21, 455 (1966).

[19]E. W. Prohofsky, Phys. Rev. 134, A1302 (1964).

[20]J. Yamashita, and K. Nakamura, Proceedings of the International Conference on Semiconductors Moscow, 1968, Vol. II, p. 942 (unpublished).

[21]R. C. Casella, Phys. Rev. Letters 5, 371 (1960); Phys. Rev. 114, 1574 (1959); J. J. Hopfield, J. Appl. Phys. Suppl. 32, 2277 (1961).

[22]D. Matz, J. Phys. Chem. Solids 28, 373 (1967).

[23]I. M. Dykman and P. M. Tomchuk, Fiz. Tverd. Tela 8, 1343 (1966) [Soviet Phys. Solid State 8, 1075 (1967)].

[24]Reference 6, p. 219.

[25]J. Kolodziejaczak, Phys. Status Solidi 19, 231 (1967).

[26]Reference 6, p. 108.

[27]J. Yamashita and N. Watanabe, Progr. Theoret. Phys. (Kyoto) 12, 443 (1954).

[28]H. J. G. Meijer and D. Polder, Physica 19, 255 (1953).

[29]By effective velocity we do not mean the average velocity over the distribution function. Instead, it implies the velocity of a single carrier obeying nonparabolic energy-momentum relationship.

[30]H. Brooks, Advan. Electron. Electron Phys. 7, 85 (1955).